

Time-frequency characterization of a sound propagation channel as an educational tool

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Abstract—This paper discusses the use of sound waves to illustrate multipath radio propagation concepts. Specifically, a procedure is presented to measure the time-varying frequency response of the channel. This helps demonstrate how a propagation channel can be characterized in time and frequency, and provides visualizations of the concepts of coherence time and coherence bandwidth. The measurements are very simple to carry out, and the required equipment is easily available. The proposed method can be useful for wireless or mobile communication courses.

I. INTRODUCTION

The study of time and frequency characteristics of multipath channels has become increasingly important in recent years, owing to the widespread use of Orthogonal Frequency-Division Multiplexing (OFDM) in wireless systems. Examples of standards that employ this technique include LTE, WiMAX and DVB-T [1, chapter 4]. OFDM organizes transmissions in the form of disjoint time and frequency resources, and deals with multipath fading variations in both domains by means of advanced techniques such as fast link-adaptation and channel-dependent scheduling [1, chapter 7]. In order to grasp how these systems work, and how they are affected by multipath propagation, it is therefore essential to characterize the time and frequency behaviour of propagation channels.

The understanding of this topic is sometimes difficult for the students, partly because of its abstract nature. It also often turns out to be difficult to see how a time-frequency grid can meaningfully arise from a single time axis (the answer lies, of course, in the fact that two different time scales are involved). Therefore, teaching of these matters can greatly benefit from the use of measurements that can illustrate the concepts. Since radio equipment tends to be expensive and bulky, specially for classroom use, sound transmitting and receiving equipment has proved to be a useful substitute, based on the existing analogy between radio and sound wave propagation [2].

A procedure for time-variation and time-dispersion measurements using audio signals has already been presented in [2]. That work essentially describes how to measure the time-varying impulse response of the channel, $h(\tau, t)$, from which the main features associated with multipath propagation can be observed. In this paper, a similar method is proposed to obtain a time-frequency characterization of the channel, by measuring its time-varying frequency response $H(f, t)$. Of course, since this function is the Fourier transform of $h(\tau, t)$ with respect to τ [3, chapter 2], a possible method would be

to estimate $h(\tau, t)$ and from it compute $H(f, t)$. However, it is more interesting and instructive to carry out a direct time-frequency measurement of the channel to obtain $H(f, t)$.

Section II briefly describes the necessary equipment. Section III presents the measurement procedure, by discussing the structure of the transmitted signal and the receiving processing, and identifies restrictions on parameters. In Section IV example measurements are shown for a suitable choice of parameter values. Conclusions are drawn in Section V.

II. NECESSARY EQUIPMENT

The basis of any channel measurement is to transmit a known signal, observe the received signal, and compare both to infer information of the channel behaviour. It is important, however, to suitably choose the transmitted signal so that the relevant features of the channel can be more easily observed.

The necessary equipment is the same as in [2], and consists of a computer with loudspeakers, which acts as the transmitter, and a second computer with a microphone, which acts as the receiver. Two different computers are preferred to prevent unwanted coupling from transmitter to receiver that sometimes occurs within a sound card. Use of the same computer for both tasks is possible if the coupling is known to be sufficiently low. Average-quality loudspeakers and microphones are perfectly adequate.

Signal processing at both transmitter and receiver is carried out in Matlab.

III. MEASUREMENT PROCEDURE. CHOICE OF TRANSMITTED SIGNAL

A. Signal structure and receiving principle

If the purpose of the measurement were to obtain $H(f_0, t)$ for a given f_0 , the simplest method would be to transmit a sinusoid at that frequency,¹ $\tilde{s}(t) = \cos(2\pi f_0 t)$, and record the received signal $\tilde{r}(t)$. In that case, the amplitude and phase variations of $\tilde{r}(t)$ as a function of time represent the values of $H(f_0, t)$. Of course, it only makes sense to speak of time variations in the amplitude and phase of $\tilde{r}(t)$ if the channel variations that cause them are slow in comparison with the period of $\tilde{s}(t)$. That is, the channel coherence time t_c should be much larger than $1/f_0$.

¹Bandpass signals are denoted with a tilde (“~”) throughout the paper.

The received signal is processed with a narrow-band filter to remove as much noise as possible. The filter bandwidth B_f should be large enough to leave the received signal with its time variations unaffected. Since time variations are seen in the frequency domain as a Doppler spread of the order of $1/t_c$, the condition for the filter bandwidth is $B_f \gg 1/t_c$.

From the above, in order to measure $H(f, t)$ at different frequencies it suffices to simultaneously transmit N sinusoids, with frequencies f_0, \dots, f_{N-1} . This allows the estimation of $H(f, t)$ sampled at those frequencies. The natural choice is evenly spaced frequencies, i.e.

$$f_i = f_0 + i\Delta f, \quad i = 0, \dots, N-1 \quad (1)$$

for a given separation Δf . This separation should be much smaller than the channel coherence bandwidth so that the frequency-domain sampling is adequate. Each carrier i is received by a narrow-band filter tuned to f_i . In this multi-carrier setting, each receiving filter serves to eliminate not only noise but also interference from the remaining carriers. To achieve good interference rejection it is sufficient that $\Delta f \gg B_f$.

Using several carriers to obtain more information about the channel comes at a price. Namely, receiving sensitivity is degraded because of the two following factors:

- 1) The total transmitted power is shared among the N carriers. This causes a loss of $10\log N$ dB per carrier.
- 2) Since the transmitted power limitation affects the peak power, not the average power, there is an additional reduction given by the peak-to-average power ratio (PAPR) of the compound signal.

Whereas the first loss is fixed and unavoidable, the second depends on the amplitudes and phases of the transmitted carriers, and thus can be diminished by a proper choice of those parameters. This is elaborated on in the following subsection.

B. Choice of subcarrier phases to minimize PAPR

The PAPR of a bandpass signal $\tilde{s}(t)$, with carrier frequency f_c , is defined as the peak power of $\tilde{s}(t)$ averaged over a carrier period divided by the average power over the whole signal duration. With this definition, the PAPR of a sinusoid is 0 dB,² and any sum of sinusoids will have a PAPR greater than this value.

Consider

$$\tilde{s}(t) = \sum_{i=0}^{N-1} A_i \cos(2\pi f_i t + \theta_i) \quad (2)$$

as argued in Subsection III-A, where A_i and θ_i are arbitrary amplitude and phases and f_i is given by (1). The choice of equal amplitudes and equal phases gives $\text{PAPR} = 10\log N$ dB, whereas other choices may give lower values. Thus appropriate amplitude and phases should be selected to achieve PAPR values as low as possible. In that case, the amplitude and

²The PAPR can also be defined considering in the numerator the maximum instantaneous power, not the maximum power averaged over a carrier period. This simply adds 3 dB to the PAPR as previously defined.

TABLE I
BEST PAPR FOR SEVERAL VALUES OF N

No. of carriers, N	Best PAPR found
21	2.7 dB
31	3.3 dB
41	3.8 dB
51	4.0 dB

phase differences between carriers need to be compensated for at the receiver. The idea is similar to that used in the SC-FDMA (Single-Carrier Frequency-Division Multiple Access) transmission scheme of the LTE uplink, which applies linear precoding to the subcarrier values in order to reduce envelope variations [1, section 5.3].

The PAPR can be expressed more conveniently in terms of lowpass equivalent signals. Assuming N odd for simplicity, the signal $\tilde{s}(t)$ given by (2) can be written as $\text{Re}[s(t) \exp j2\pi f_c t]$, with $f_c = f_{(N-1)/2}$ and

$$s(t) = \sum_{i=0}^{N-1} A_i \exp \left(j \left(2\pi \Delta f \left(i - \frac{N-1}{2} \right) t + \theta_i \right) \right). \quad (3)$$

The lowpass equivalent signal $s(t)$ is a sum of sinusoids of frequencies $(-N+1)\Delta f/2, (-N+3)\Delta f/2, \dots, (N-1)\Delta f/2$. Since N is odd, all these values are integer multiples of Δf , and therefore $s(t)$ is periodic with period $1/\Delta f$. The power of $\tilde{s}(t)$ averaged over a carrier period is $|s(t)|^2/2$, and thus the PAPR of $\tilde{s}(t)$ is given by

$$\text{PAPR} = \frac{\max_{t \in (0, 1/\Delta f)} |s(t)|^2}{\text{mean}_{t \in (0, 1/\Delta f)} |s(t)|^2} = \frac{\max_{t \in (0, 1/\Delta f)} |s(t)|^2}{\Delta f \int_0^{1/\Delta f} |s(t)|^2 dt}. \quad (4)$$

For want of an analytic method, a possible approach to the problem of minimizing the PAPR is to test a large number of randomly generated combinations of A_i and θ_i , and choose that which gives the lowest value computed from (4). Experimentally it has been found that approximately the same minimum PAPR is obtained if the amplitudes are restricted to be equal and only the phases are allowed to vary. This has the advantage that the receiving sensitivity is the same for all frequencies (otherwise, carriers transmitted with lower amplitudes would have to be amplified at the receiver, which would cause an increment of noise power and thus degrade sensitivity).

As stems from the previous analysis, the optimum combination of N phases does not depend on Δf , but only on N . Table I shows the best PAPR obtained from testing $5 \cdot 10^6$ phase combinations in each case. It is seen that the resulting PAPR values are quite low.

As an example, one period of $s(t)$ for $N = 41$ is depicted in Figure 1. The low PAPR is evident from the small differences in lobe amplitudes.

C. Further processing details. Restrictions on parameters

All signals are discrete-time. The sampling rate f_s should be much larger than $f_{N-1} = f_0 + (N-1)\Delta f$ to ensure that the sampling is adequate.

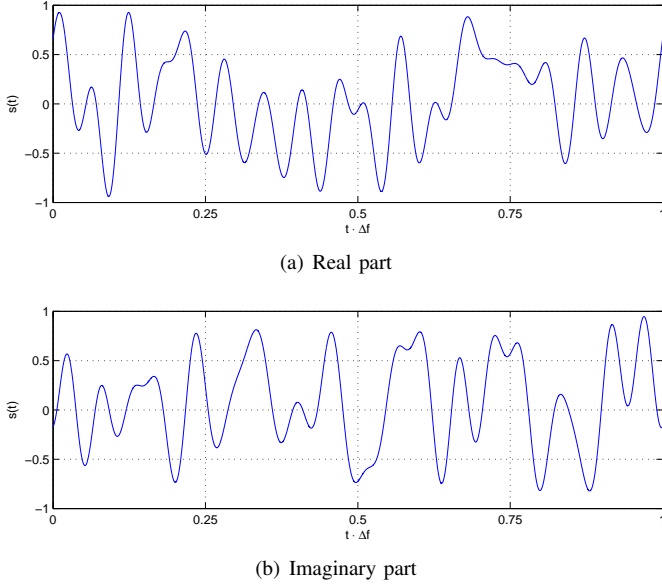


Fig. 1. Signal $s(t)$ for $N = 41$

Using two different computers for transmitting and receiving raises the issue of clock rate discrepancies. These are typically of the order of several tens of ppm [2]. This effect is important for the phase of $H(f, t)$, in which it introduces a constant drift. It must be corrected by previously estimating the clock rate difference between both computers. As for the magnitude of $H(f, t)$, however, the clock rate difference can be ignored, because the frequency drift that it causes is negligible compared with the filter bandwidth B_f .

The frequency responses of the narrow-band receiver filters should be selected to meet the following conditions:

- 1) Steep fall in the transition from passband to rejection band.
- 2) Passband as wide as possible, to allow measuring fast channel variations. Besides, the filter response should be approximately constant over the passband.

These conditions are partly opposing, and thus a tradeoff is necessary. A good compromise has been found to be a Butterworth filter of order 8 and 3-dB passband given as $f_i \pm 0.3\Delta f$. In the following, $\tilde{r}_i(t)$ will denote the output signal of the i -th filter.

Although the measurement provides an estimation of both amplitude and phase of $H(f, t)$, in the sequel only the amplitude will be considered. The amplitude of $H(f_i, t)$ is proportional to that of $\tilde{r}_i(t)$, i.e. $|H(f_i, t)|$ is represented by the envelope of $\tilde{r}_i(t)$. To extract this information, the same envelope detector as in [2] can be used, namely a lowpass filter applied to the absolute value of $\tilde{r}_i(t)$. The cutoff frequency f_e of the envelope filter should satisfy $B_f \ll f_e \ll f_0$. A Butterworth design of order 4 with cutoff (3-dB) frequency 40 Hz is used.

The magnitude of $H(f, t)$ is displayed as an image, and is best represented in dB normalized to its maximum. The interference between carriers and the noise level impose a

TABLE II
RESTRICTIONS ON PARAMETERS

I	Slowly varying channel	$1/f_0 \ll t_c$
II	Receiver filter frequency resolution	$B_f \ll \Delta f$
III	Receiver filter time resolution	$1/B_f \ll t_c$
IV	Frequency sampling	$\Delta f \ll B_c$
V	Time sampling	$f_0 + (N-1)\Delta f \ll f_s$
VI	Envelope detector filter	$B_f \ll f_e \ll f_0$

limitation on the values that can be represented, i.e. the representation range for $|H(f, t)|$ should be sufficiently small that those effects are not noticeable.

Given a signal duration D , the image will initially consist of N points in frequency by $f_s D$ points in time. For N as in Table I, interpolation in frequency should be applied in order to obtain a smooth image; whereas for most values of f_s and D the number of points in time needs to be reduced by downsampling.

The parameter restrictions that have been identified along this section are summarized in Table II.

IV. EXAMPLE MEASUREMENTS

A. Parameter selection

For a moderate-size room, a time dispersion of tens of ms is to be expected [2]. This corresponds to a coherence bandwidth of a few tens of Hz. In order to adequately observe the frequency selectivity of the channel (restriction IV), $\Delta f = 2$ Hz is chosen. Taking $N = 41$ gives a frequency span of 80 Hz.

A low value of f_0 makes it easier to fulfill restriction I, and also III, because t_c is inversely proportional to f_0 . In any case, movement of the microphone should be sufficiently slow to meet these conditions. In the examples to follow $f_0 = 560$ Hz is used, i.e. the frequency interval 560–640 Hz is measured. The signal duration is set to 20 s.

Restriction V can be satisfied with relatively low sample frequencies. This helps keep file size low and receiver processing fast. A value of 12000 Hz is selected for f_s .

The parameters for the receiving filters and envelope detector have already been discussed in Subsection III-C.

B. Measurements

All measurements have been taken in an indoor environment, with the parameter values given in Subsection IV-A. The receiving bandpass filters, with $B_f = 1.2$ Hz, have a step response that takes roughly 1 s to stabilize. In order to remove ringing or other transient effects at the beginning and end of the received signal, only a central section of 18 s is represented.

Figure 2 shows the measurements in a small room where microphone, loudspeakers and all other objects remain still. The channel response is seen to remain constant with time, and a coherence bandwidth of a few tens of Hz is observed. Figure 3 is a similar measurement in a larger room. The presence of larger delays gives rise to more pronounced frequency selectivity in this case, and thus the coherence bandwidth is smaller.

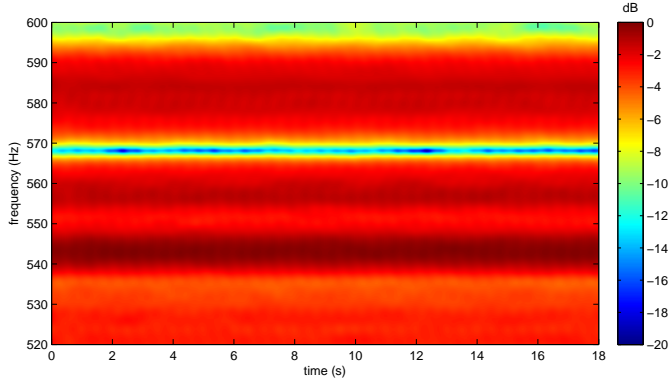


Fig. 2. Static environment, small room

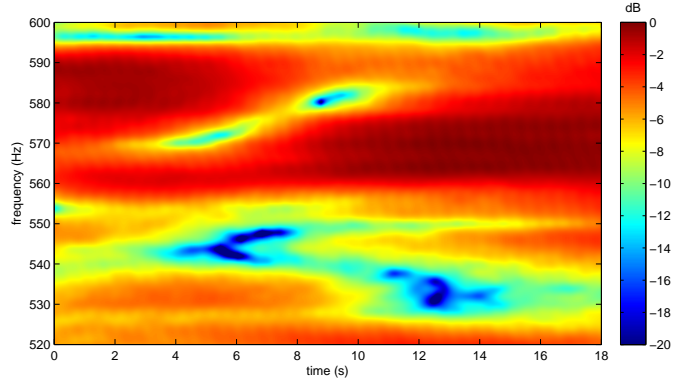


Fig. 5. Moving microphone, slowly

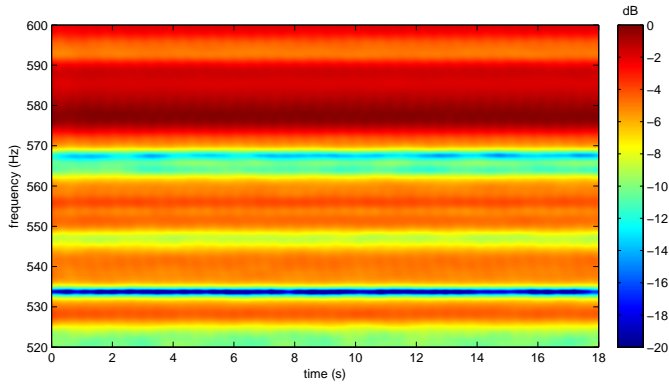


Fig. 3. Static environment, larger room

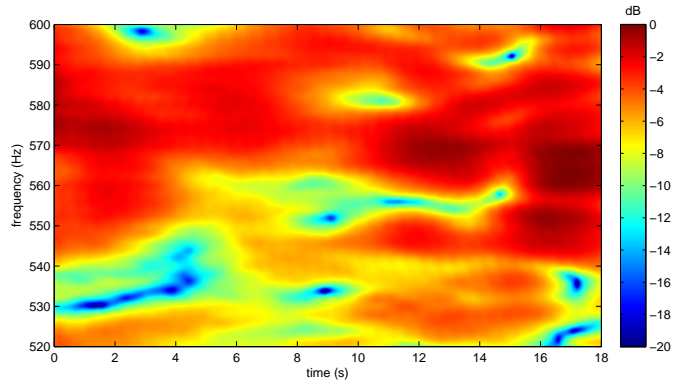


Fig. 6. Moving microphone, slightly faster

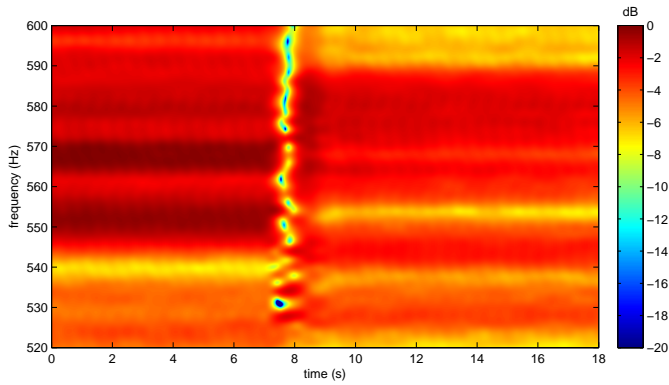


Fig. 4. Abrupt change in microphone placement

The transient response of the filters is illustrated in Figure 4. In this measurement the microphone is held still for a few seconds, then it is suddenly moved and turned, and after that it is left in the new position and orientation until the measurement ends. The figure shows the two different channel responses, separated by a stabilization period of the order of 1 s, as expected.

Figures 5 and 6 correspond to gradually varying channels. The room is the same as in Figure 2, but the microphone is continuously moved; very slowly in Figure 5, and slightly

faster in Figure 6. This is reflected in the time variation seen in the figures, with a smaller coherence time in the second measurement. Note that in both cases channel variations are slow compared to $1/B_f$ (restriction II). The figures also make evident the potential benefits of techniques such as channel-dependent scheduling and link adaptation, which exploit channel maxima in time and frequency.

V. CONCLUSIONS

This paper extends the idea introduced in [2], of using sound waves to illustrate multipath propagation concepts, to the frequency domain. The measurements allow the students to observe the time variation and frequency selectivity of a propagation channel, and to visualize the concepts of coherence time and coherence bandwidth associated with multipath fading. The measurement procedure is simple and relies on basic equipment. The proposed method can be applied to enhance the teaching of courses on mobile or wireless communications.

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